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Problem Set 10

Quantum Mechanics & Atomic Structure

Problem 1. *At what rate does the Sun emit photons? For simplicity, assume that the Sun's entire emission at the rate of* $3.9 \times 10^{26}\,\mathrm{W}$ *is at the single wavelength of* $550\,\mathrm{nm}$.

Solution. Let R be the rate of photon emission (number of photons emitted per unit time) of the Sun and let E be the energy of a single photon. Then the power output of the Sun is given by $P = RE$. Now

$$E = hf = hc/\lambda$$

where $h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$ is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and

$$R = \frac{\lambda P}{hc} = \frac{(550\,\mathrm{nm})(3.9 \times 10^{26}\,\mathrm{W})}{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})(2.998 \times 10^{8}\,\mathrm{m/s})} = 1.0 \times 10^{45}\,\mathrm{photons/s}$$

□

Problem 2. *A* 100 W *sodium lamp* ($\lambda = 589\,\text{nm}$) *radiates energy uniformly in all directions.* ***(a)*** *At what rate are photons emitted by the lamp?* ***(b)*** *At what distance from the lamp will a totally absorbing screen absorb photons at the rate of* $1.00\,\text{photon/cm}^2{\cdot}\text{s}$*?* ***(c)*** *What is the photon flux (photons per unit area per unit time) on a small screen* $2.00\,\text{m}$ *from the lamp?*

Solution. **(a)** We assume all the power results in photon production at the wavelength $\lambda = 589\,\text{nm}$. Let R be the rate of photon production and E be the energy of a single photon. Then,

$$P = RE = Rhc/\lambda$$

where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{(589 \times 10^{-9}\,\text{m})(100\,\text{W})}{(6.63 \times 10^{-34}\,\text{J} \cdot \text{s})(3.00 \times 10^{8}\,\text{m/s})} = 2.96 \times 10^{20}\,\text{photon/s}$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96 \times 10^{20}\,\text{photon/s}}{4\pi(1.00 \times 10^{4}\,\text{photon/m}^2{\cdot}\text{s})}} = 4.86 \times 10^{7}\,\text{m}$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96 \times 10^{20}\,\text{photon/s}}{4\pi(2.00\,\text{m})^2} = 5.89 \times 10^{18}\,\frac{\text{photon}}{\text{m}^2{\cdot}\text{s}}$$

□

Problem 3. *A satellite in Earth orbit maintains a panel of solar cells of area* $2.60\,\mathrm{m}^2$ *perpendicular to the direction of the Sun's light rays. The intensity of the light at the panel is* $1.39\,\mathrm{kW/m}^2$. ***(a)*** *At what rate does solar energy arrive at the panel?* ***(b)*** *At what rate are solar photons absorbed by the panel? Assume that the solar radiation is monochromatic, with a wavelength of* $550\,\mathrm{nm}$, *and that all the solar radiation striking the panel is absorbed.* ***(c)*** *How long would it take for a "mole of photons" to be absorbed by the panel?*

Solution. **(a)** The rate at which solar energy strikes the panel is

$$P = (1.39\,\mathrm{kW/m}^2)(2.60\,\mathrm{m}^2) = 3.61\,\mathrm{kW}$$

(b) The rate at which solar photons are absorbed by the panel is

$$\begin{aligned} R &= \frac{P}{E_{ph}} = \frac{3.61 \times 10^3\,\mathrm{W}}{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})(2.998 \times 10^8\,\mathrm{m/s})/(550 \times 10^{-9}\,\mathrm{m})} \\ &= 1.00 \times 10^{22}\,\mathrm{photon/s} \end{aligned}$$

(c) The time in question is given by

$$t = \frac{N_A}{R} = \frac{6.02 \times 10^{23}}{1.00 \times 10^{22}\,/\mathrm{s}} = 60.2\mathrm{s}$$

□

Problem 4. *Light of wavelength* 200 nm *shines on an aluminum surface;* 4.20 eV *is required to eject an electron. What is the kinetic energy of **(a)** the fastest and **(b)** the slowest ejected electrons? **(c)** What is the stopping potential for this situation? **(d)** What is the cutoff wavelength for aluminum?*

Solution. The kinetic energy K_m of the fastest electron emitted is given by

$$K_m = hf - A$$

where A is the work function of aluminum, and f is the frequency of the incident radiation. Since $f = c/\lambda$, where λ is the photon wavelength, the above expression can be rewritten as

$$K_m = (hc/\lambda) - A$$

(a) Thus, the kinetic energy of the fastest electron is

$$K_m = \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{200\,\mathrm{nm}} - 4.20\,\mathrm{eV} = 2.00\,\mathrm{eV}$$

where we have used $hc = 1240\,\mathrm{eV}\cdot\mathrm{nm}$.

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_{stop} is given by $K_m = eV_{\mathrm{stop}}$, so

$$V_{\mathrm{stop}} = K_m/e = (2.00\,\mathrm{eV})/e = 2.00\,\mathrm{V}$$

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus, $hc/\lambda_0 = A$ or

$$\lambda_0 = hc/A = (1240\,\mathrm{eV}\cdot\mathrm{nm})/(4.2\,\mathrm{eV}) = 295\,\mathrm{nm}$$

□

Problem 5. *The stopping potential for electrons emitted from a surface illuminated by light of wavelength* 491 nm *is* 0.710 V*. When the incident wavelength is changed to a new value, the stopping potential is* 1.43 V*.* ***(a)*** *What is this new wavelength?* ***(b)*** *What is the work function for the surface?*

Solution. **(a)** We use the photoelectric effect equation in the form $hc/\lambda = A + K_m$. The work function depends only on the material and the condition of the surface, and not on the wavelength of the incident light. Let λ_1 be the first wavelength described and λ_2 be the second. Let $K_{m1} = 0.710\,\mathrm{eV}$ be the maximum kinetic energy of electrons ejected by light with the first wavelength, and $K_{m2} = 1.43\,\mathrm{eV}$ be the maximum kinetic energy of electrons ejected by light with the second wavelength. Then,

$$hc/\lambda_1 = A + K_{m1}, \qquad hc/\lambda_2 = A + K_{m2}$$

The first equation yields $A = (hc/\lambda_1) - K_{m1}$. When this is used to substitute for A in the second equation, the result is

$$(hc/\lambda_2) = (hc/\lambda_1) - K_{m1} + K_{m2}$$

The solution for λ_2 is

$$\lambda_2 = \frac{hc\lambda_1}{hc + \lambda_1(K_{m2} - K_{m1})} = \frac{(1240\,\mathrm{eV}\cdot\mathrm{nm})(491\,\mathrm{nm})}{1240\,\mathrm{eV}\cdot\mathrm{nm} + (491\,\mathrm{nm})(1.43\,\mathrm{eV} - 0.710\,\mathrm{eV})}$$
$$= 382\,\mathrm{nm}$$

Here $hc = 1240\,\mathrm{eV}\cdot\mathrm{nm}$ has been used.

(b) The first equation displayed above yields

$$A = \frac{hc}{\lambda_1} - K_{m1} = \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{491\,\mathrm{nm}} - 0.710\,\mathrm{eV} = 1.82\,\mathrm{eV}$$

□

Problem 6. *Singly charged sodium ions are accelerated through a potential difference of* $300\,\mathrm{V}$*.* ***(a)*** *What is the momentum acquired by such an ion?* ***(b)*** *What is its de Broglie wavelength? (Hint: The mass of a single sodium atom is* $3.819 \times 10^{-26}\,\mathrm{kg}$*)*

Solution. The de Broglie wavelength of the sodium ion is given by $\lambda = h/p$, where p is the momentum of the ion. The kinetic energy acquired is $K = qU$, where q is the charge on an ion and U is the accelerating potential. Thus, the momentum of an ion is $p = \sqrt{2mK}$, and the corresponding de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$.

(a) The kinetic energy of the ion is

$$K = qU = (1.60 \times 10^{-19}\,\mathrm{C})(300\,\mathrm{V}) = 4.80 \times 10^{-17}\,\mathrm{J}$$

Thus, the momentum of a sodium ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819 \times 10^{-26}\,\mathrm{kg})(4.80 \times 10^{-17}\,\mathrm{J})} = 1.91 \times 10^{-21}\,\mathrm{kg \cdot m/s}$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{1.91 \times 10^{-21}\,\mathrm{kg \cdot m/s}} = 3.46 \times 10^{-13}\,\mathrm{m}$$

On this basis, we know the greater the potential difference, the greater the kinetic energy and momentum, and hence, the smaller the de Broglie wavelength. □

Problem 7. *What is the wavelength of* ***(a)*** *a photon with energy* $1.00\,\mathrm{eV}$*,* ***(b)*** *an electron with energy* $1.00\,\mathrm{eV}$*,* ***(c)*** *a photon of energy* $1.00\,\mathrm{GeV}$*, and* ***(d)*** *an electron with energy* $1.00\,\mathrm{GeV}$*?*

Solution. **(a)** The momentum of the photon is given by $p = E/c$, where E is its energy. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{1.00\,\mathrm{eV}} = 1240\,\mathrm{nm}$$

(b) The momentum of the electron is given by $p = \sqrt{2mK}$, where K is its kinetic energy and m is its mass. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\,\mathrm{J}\cdot\mathrm{s}}{\sqrt{2(9.109\times10^{-31}\,\mathrm{kg})(1.602\times10^{-19}\,\mathrm{J/eV})K}} = \frac{1.226\times10^{-9}\,\mathrm{m}\cdot\mathrm{eV}^{1/2}}{\sqrt{K}} = \frac{1.226\,\mathrm{nm}\cdot\mathrm{eV}^{1/2}}{\sqrt{K}}$$

For $K = 1.00\,\mathrm{eV}$, we have

$$\lambda = \frac{1.226\,\mathrm{nm}\cdot\mathrm{eV}^{1/2}}{\sqrt{1.00\,\mathrm{eV}}} = 1.23\,\mathrm{nm}$$

(c) For the photon,

$$\lambda = \frac{hc}{E} = \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{1.00\times10^{9}\,\mathrm{eV}} = 1.24\times10^{-6}\,\mathrm{nm} = 1.24\,\mathrm{fm}$$

(d) Relativity theory must be used to calculate the wavelength for the electron. According to relativity theory, the momentum p and kinetic energy K are related by

$$(pc)^2 = K^2 + 2Kmc^2$$

Thus,

$$\begin{aligned} pc &= \sqrt{K^2 + 2Kmc^2} = \sqrt{(1.00\times10^{9}\,\mathrm{eV})^2 + 2(1.00\times10^{9}\,\mathrm{eV})(0.511\times10^{6}\,\mathrm{eV})} \\ &= 1.00\times10^{9}\,\mathrm{eV} \end{aligned}$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{1.00\times10^{9}\,\mathrm{eV}} = 1.24\times10^{-6}\,\mathrm{nm} = 1.24\,\mathrm{fm}$$

□

Problem 8. *An electron in a multielectron atom is known to have the quantum number $\ell = 3$. What are its possible n, m_ℓ, and m_s quantum numbers?*

Solution. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values $-3, -2, -1,\ 0,\ +1,\ +2,$ or $+3$. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$. $\square$

Problem 9. *Two of the three electrons in a lithium atom have quantum numbers* (n, ℓ, m_ℓ, m_s) *of and* $(1, 0, 0, +\frac{1}{2})$, *and* $(1, 0, 0, -\frac{1}{2})$. *What quantum numbers are possible for the third electron if the atom is* ***(a)*** *in the ground state and* ***(b)*** *in the first excited state?*

Solution. The four quantum numbers (n, ℓ, m_ℓ, m_s) identify the quantum states of individual electrons in a multi-electron atom. A lithium atom has three electrons. The first two electrons have quantum numbers $(1, 0, 0, \pm 1/2)$. All states with principal quantum number $n = 1$ are filled. The next lowest states have $n = 2$.

The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same.

(a) Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}$ or $n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}$. That is, $(n, \ell, m_\ell, m_s) = (2, 0, 0, +1/2)$ and $(2, 0, 0, -1/2)$.

(b) The next lowest state in energy is an $n = 2, \ell = 1$ state. All $n = 3$ states are higher in energy. The magnetic quantum number can be $m_\ell = -1, 0, \text{or} +1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Thus, $(n, \ell, m_\ell, m_s) =$ (2,1,1,+1/2), $(2, 1, 1, -1/2)$, (2,1,0,+1/2), $(2, 1, 0, -1/2)$, (2,1,1,+1/2), $(2, 1, 1, -1/2)$.

DISCUSSION No two electrons can have the same set of quantum numbers, as required by the Pauli exclusion principle. □

Problem 10. *Show that the number of states with the same quantum number n is $2n^2$.*

Solution. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to $n-1$. For any value of ℓ, there are $2\ell+1$ possible values of the magnetic quantum number m_ℓ, ranging from to $-\ell$ to ℓ. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s = -\frac{1}{2}$ and the other corresponding to $m_s = +\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N = 2\sum_{\ell=0}^{n-1}(2\ell+1)$$

Now

$$\sum_{\ell=0}^{n-1} 2\ell = 2\sum_{\ell=0}^{n-1} \ell = 2\frac{n}{2}(n-1) = n(n-1)$$

since there are n terms in the sum and the average term is $(n–1)/2$. Furthermore,

$$\sum_{\ell=0}^{n-1} 1 = n$$

Thus, $N = 2n(n-1) + n = 2n^2$. $\square$